An early example of constraint propagation

1 Introduction

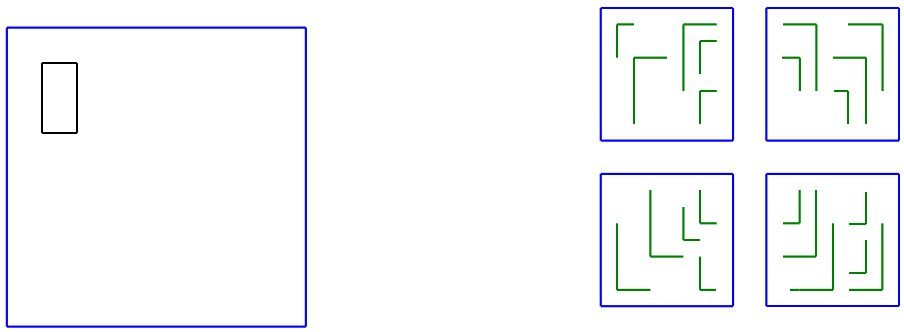
This document is intended to provide a brief easy-to-read introduction to a paper entitled *Parallel Recognition of Idealised Line Characters*, by J.R. Ullmann, published in *Kybernetik*, Vol 2, Part 5, June 1965, pp 221-226. This work is not claimed to be practical; instead it is an exploration of basic ideas, raising questions that remain unanswered after more than 40 years.

In this simplified introduction, we attempt to recognize only one pattern, which we call the *target*. We confine our attention to closed polygons in which the angle at each vertex is 90 or 270 degrees. A pattern (in this context) is a polygon in which lines are composed of 1's on a background of 0's in a one-dimensional array of bits which we call the *retina*. Actually this work is not concerned with individual bits. And, for simplicity, the target never touches the edge of the retina. Also, for simplicity, we are concerned only with line drawings: a line does not have a black side and a white side.

The aim of this (simplified) work is to recognise absolutely *any* given target polygon on the retina, regardless of the presence of other patterns on the retina. The target is to be recognized even when it is shifted into different positions and/or scaled to different sizes.

In this work a *line pair* is a pair of lines that meet at a vertex of a polygon. Any given line pair belongs to exactly one set, L_i, of line-pairs. Within a set L_i, line pairs differ from each other only in location and/or in size (but not in rotation). Here *location* means location on the retina. If a set L_i contains a given line-pair λ, then L_i certainly includes all possible shifted and differently-sized copies of λ.

We define $H = \{L_k|$ at least one line pair in L_k is present on the retina when the target is present on the retina$\}$. Henceforward, *present* means *present on the retina*. The following paragraphs introduce three schemes for deciding whether or not the target is present. It is easy to see that constraint propagation is used in Scheme B. It is less easy to see *why* constraint propagation is actually essential in this context.



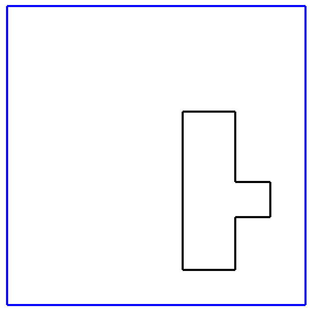
(a) (b)

Figure 1: A blue rectangle marks the edges of the retina. (a) Example of a target. (b) A few examples of line-pairs in the sets L_1, L_2, L_3 and L_4 for this target.

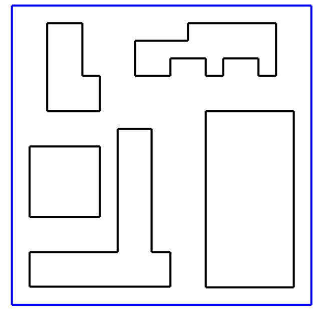
2 Scheme Z

Scheme Z is: decide that the target is present if every set $L_k \in H$ includes at least one line pair that is present; otherwise decide that the target is absent. Throughout this introductory document we always use the same example in which the target is the rectangle shown in Fig 1(a). Each of the four line-pairs in this simple target belongs to a different set L_k. Fig 1(b) shows a few members of each of the four sets L_1, L_2, L_3 and L_4. If the target is present (in any location or size), Scheme Z will obviously decide correctly that it is present, even if other patterns are present at the same time, as in Fig 2(b). For Fig 2(a) Scheme Z will decide correctly that the target is not present.

For Fig 3(a), Scheme Z incorrectly decides that the target is present, because the polygon in Fig 3(a) includes at least one line pair in each of L_1, L_2, L_3 and L_4. To prevent such errors we introduce Scheme A.



(a)

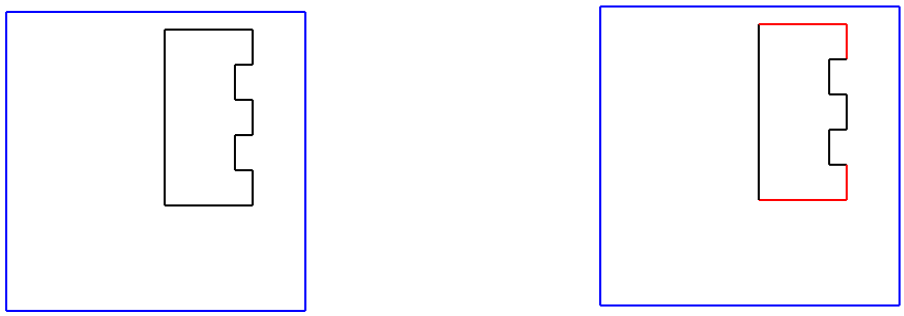


(b)

Figure 2: (a) An example of a pattern that should not be recognized as the target. (b) The target (at the bottom right) and some other patterns.

3 Scheme A

Scheme A is: decide that the target is present if every set $L_k \in H$ includes at least one line pair that is present on the retina and no set $L_h \notin H$ contains any line pair that is present on the retina; otherwise decide that the target is absent. Scheme A correctly decides that the target is present in Fig 1(a) and not present in Fig 3(a). But Scheme A incorrectly decides that the target is absent from Fig 2(b) because, although the target is actually present, other patterns (i.e. polygons) are also present. To prevent such errors we introduce Scheme B.



(a) (b)

Figure 3: (a) Every line-pair in the target is included in this shape. (b) The red line-pairs are not in W_0 because they are not in the target.

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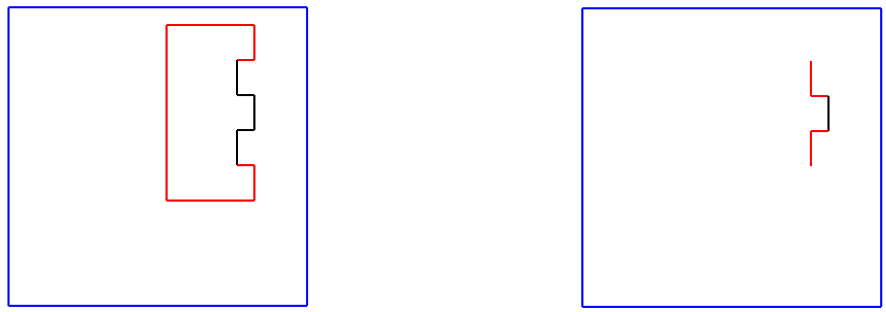
4 Scheme B

To formulate Scheme B we say that a line pair λ is *plausible* iff λ belongs to any set $L_k \in H$ and every set $L_g \in H$ includes at least one line pair that is present. And we define $W_0 = \{\lambda | \lambda \text{ is present and plausible}\}$. Scheme B is:

begin
  compute W_0;
  repeat
    $i := i + 1$;
    $W_i := \{\lambda|$ each line in λ belongs to two plausible pairs in $W_{i-1}\}$
    delete every line that does not belong to at least two line pairs in W_i;
  until $W_i = W_{i-1}$;
  decide that the target is present if every set $L_k \in H$ now includes at least one line pair that is present; otherwise decide that the target is absent.
end;

Scheme B correctly recognizes that the target is present in Fig 1(a), absent in Fig 2(a) and present in Fig 2(b). For Fig 3(a), Scheme B successively deletes unsupported pairs as shown in Figs 3(b), 4(a) and 4(b). W_3 is empty, so Scheme B correctly decides that the target is absent. Fig 7 in the Kybernetic paper provides a more complex example of constraint propagation.



(a) (b)

Figure 4: (a) The red line-pairs are not in W_1 because they are not supported. (b) The red line-pairs are not in W_2 because they are not supported.

5 Related research

J.R. Ullmann, "A Use of Continuity in Character Recognition". IEEE Transactions on Systems, Man and Cybernetics, Vol SMC-4, No 3, May 1974, pp 294-300. Using binary constraint propagation, this attempts to determine whether a given character could possibly be a distorted image of another given character. Distortion preserves continuity. A mapping between the two given characters is subject to a continuity constraint implemented with corresponding pairs of line pairs.

J.R. Ullmann, "Subset Methods for Recognising Distorted Patterns". IEEE Transactions on Systems, Man and Cybernetics, Vol SMC-7, No 3, March 1977, pp 180-191. Using non-binary constraint propagation, this is a later development that does not require all boundaries to be closed.

J.R. Ullmann, "Pattern Recognition Using Degenerate Reference Data". In Pattern Recognition and Artificial Intelligence, edited by C.H.Chen, Academic Press, 1976, pp 508-528. Although this was published before "Subset Methods..." it was actually later work, in which distorted patterns were compared against reference patterns stored in a non-binary constraint network. This work provides further examples of non-binary constraint propagation.

J.R.U., January 2009